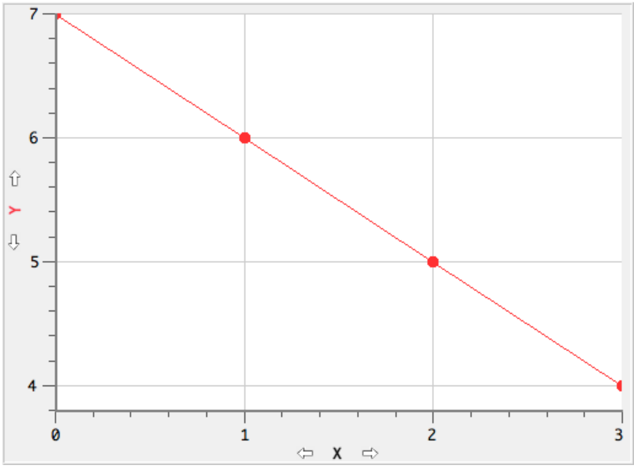
Graphing Exercises



The equation of a straight line in the *x-y* plane is given as follows:

$$y = mx + b$$

where *y* is the value of a point on the *y*-axis, *x* is the value of a point on the *x*-axis, *m* is the slope of the line, and *b* is the value of *y* when *x=0. b* is also known as the *y*-intercept (this is a hyphenated word and does NOT mean "y minus intercept"; *m* is defined in as follows:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

From the above graph we derive the values of *m* (choosing any two points on the best-fit straight line) and *b* (the value of *y* when *x=0*). Here we choose two points as follows:

Point 1: $(x_1, y_1) = (1, 6)$
Point 2: $(x_2, y_2) = (2, 5)$

$$m = \frac{5-6}{2-1} = \frac{-1}{1} = -1$$

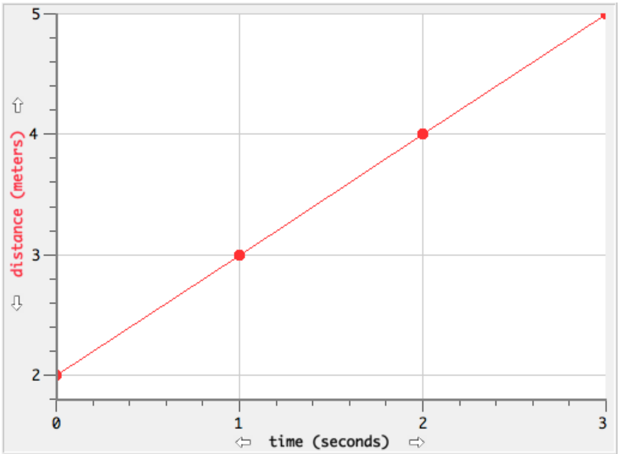
By inspection, the value of the y-intercept, *b* (the y value when x = 0), is 7.

Hence, the relationship between y and x in the above diagram is given by the relationship y=mx+b.

$$\begin{aligned} & y = mx + b \\ \text{That is, } & y = -1x + 7 \\ & y = -x + 7 \end{aligned}$$

This equation can be used to predict values of *y* (the dependent variable) when *x* (the independent variable) is known. For instance, if *x*=3, then *y*=-3+7=4. The point (*x, y*) = (3, 4) is clearly on the graph.

When real values such as time (whose units are seconds) and position (whose units are meters) are plotted, one needs to address both variables and their units. Here's an example, a plot of position (graphed on the y-axis) and time (graphed on the x-axis).



Using two points, (1, 3) and (2, 4), on the best-fit line to determine the value of the slope, m:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3m - 4m}{1s - 2s} = \frac{-1m}{-1s} = 1 \,{}^{m}\!/_{s}$$

and by inspection b=2m at time = 0s.

Hence, y=mx+b corresponds to the following (replacing y by the corresponding variable and its units and x by the corresponding variable and its units):

$$y = mx + b$$
$$\text{distance} = (1m/s) * \text{time} + 2m$$

Using this relationship, one can predict the value of distance given the time. For example, what is the distance at time = 3 seconds?

$$\text{distance} = (1m/s) * (3s) + 2m$$
$$\text{distance} = 3m + 2m$$
$$\text{distance} = 5m$$

The point (x, y) = (time, distance) = (3s, 5m) is clearly on the best-fit line.

Now, given this information, determine the relationship between variables for the following two graphs: